Two-dimensional flow response to tidal fluctuation in a heterogeneous aquifer-aquitard system

Quanrong Wang,1 Hongbin Zhan1,2* and Zhonghua Tang1

1 School of Environmental Studies, China University of Geosciences, Wuhan, Hubei, 430074, China
2 Department of Geology and Geophysics, Texas A&M University, College Station, TX, 77843-3115, USA

Abstract:

In alluvial coastal aquifers, finer sediments are preferentially deposited along the downstream direction, so the hydraulic conductivity is generally heterogeneous and changes with distance from the coastline. To investigate the influence of aquifer heterogeneity on seawater-groundwater interaction, a new two-dimensional model characterising groundwater flow in an aquifer-aquitard system was developed assuming that the hydraulic conductivity of the aquifer linearly increases with the distance from the coastline along the inland direction. A closed-form analytical solution was derived using the separation-of-variables method. Comparing the new solution with the numerical solution by COMSOL Multiphysics (Sweden) based on the finite-element method, one can see that the new solution agreed with the numerical solution very well except at the early time. We found that both aquitard leakance and the heterogeneity factor (b) could result in the propagation bias. The propagation bias represents the inconsistency between the theoretical calculation and the observed strong attenuation and small time lag between the head and tide fluctuations. The attenuation decreased with perpendicular distance from the coastline (x-axis), whereas the time lag increased with distance along the x-axis. The relationship between the time lag and the distance along the x-axis seemed to be linear when b was 0.001 m⁻¹, whereas it obeyed a power function when b was greater than 0.01 m⁻¹. Copyright © 2014 John Wiley & Sons, Ltd.

KEY WORDS coastal aquifer; groundwater flow; aquitard leakance; heterogeneity

Received 21 October 2013; Accepted 25 March 2014

INTRODUCTION

In a coastal aquifer, periodic ocean tides often serve as the boundary of groundwater flow when the aquifer is directly connected to the ocean. This tidal loading results in the periodically fluctuating hydraulic heads in the aquifer, where the fluctuation frequencies will be the same as those of the tidal fluctuations. Even when the groundwater flow in the coastal aquifer was suddenly disturbed by the human activities, such as pumping/injecting water, such an effect will die out after a certain lapse of time (Li et al., 2002). Since the 1950s, the research on the interaction between groundwater and seawater has attracted great attention among hydrogeologists, for example, Jacob (1950), Ferris (1951), Carr and Vanderka (1969), Pandit et al. (1991), Sun (1997), Li and Jiao (2001), to name a few. Jiao and Tang (1999) employed the complex transform method to change the time-dependent governing equation of groundwater flow into a time-independent equation, and then derived an analytical solution of groundwater response to tidal fluctuations in a leaky confined aquifer under the quasi steady-state condition. The quasi steady-state condition implies that the effect of the initial condition on the flow can be ignored. Li and Jiao (2001), Sun et al. (2008), Guo et al. (2010), and Asadi-Aghbolaghi et al. (2012) applied similar approaches as Jiao and Tang (1999) but to somewhat different scenarios of coastal aquifers and derived several one-dimensional analytical solutions for hydraulic head distributions.

In fact, the one-dimensional model of the groundwater flow in response to tidal fluctuations is a simplification, because the tidal propagation along the coastline may create significant phase shifts and amplitude changes, because of friction between the seawater and ocean bottom. This phenomena has been observed in many places, such as in Apalachicola Bay, Florida, USA (Sun, 1997). Sun (1997) pointed out that the two-dimensional model of groundwater flow may be more general and applicable, and he presented the corresponding analytical solutions. Li et al. (2000) derived an analytical solution of the two-dimensional model considering the interaction between the cross-shore and along-shore tidal waves in the aquifer near the estuary’s entry. Recently, Li et al. (2006) and Liu et al. (2008) proposed a semi-analytical method to simulate the two-dimensional flow in a coastal aquifer with irregular boundaries. All the studies motioned earlier on...
the two-dimensional model assumed that the aquifer was isolated from the adjacent aquitards. In fact, the flow leakage through the aquitard from the overlying unconfined aquifer may impact the amplitude attenuation and the time lag of the head fluctuations in the aquifer (Jiao and Tang, 1999; Tang and Jiao, 2001; Trefry and Bekele, 2004; Sun et al., 2008). Huang et al. (2012) pointed out that the vertical flow through aquitard to the adjacent confined aquifers cannot be ignored even if the aquitard permeability was much lower than that of the aquifer. Subsequently, Tang and Jiao (2001) extended the model of Sun (1997) by considering the aquitard leakance for the two-dimensional flow.

The aforementioned studies on tidal fluctuations assumed that the aquifer was homogeneous and isotropic. However, many studies reported that the hydraulic properties of the coastal aquifers exhibit heterogeneity and anisotropy (Trefry and Bekele, 2004; Carol et al., 2009; Cardenas, 2010). For instance, Guo et al. (2010) found that a coastal aquifer, in the western part of Dongzhai Harbor, Hainan Province of China, cannot be treated as homogeneous, based on the analysis of borehole data and the slug test. Carol et al. (2009) pointed out that the hydraulic conductivity increased with distance from the coastline in a coastal plain of the south Samborombon Bay wetland of Argentina. Montalto et al. (2006) concluded that the aquifer conductivity linearly changed with distance from the coastline in the Hudson River estuary, USA. In fact, the aforementioned observations were reasonable for many alluvial coastal aquifers, because finer sediments are preferentially deposited along the downstream direction (Lunt et al., 2004; Cardenas, 2010; Chuang et al., 2010).

Because of the horizontal heterogeneity of the aquifer, the governing equations are difficult to solve analytically. A review of existing publications on groundwater flow in coastal heterogeneous aquifer indicated that two types of approaches have been employed to handle such problems. The first approach is referred to herein as the sub-region model. This model assumed that the heterogeneous aquifer could be divided into many horizontal homogeneous sub-regions. Examples of studies using the sub-region model approach follows. Guo et al. (2010) employed the two-region model to successfully fit the observed data in a heterogeneous coastal aquifer in the west part of Dongzhai Harbor, Hainan Province of China. Chuang et al. (2010) derived analytical solutions of groundwater flow in a coastal aquifer with a finite number of horizontal sub-regions. Trefry (1999) presented the algebraic solutions for tidal propagation in a spatially heterogeneous coastal aquifer, where the heterogeneity was approximated by a finite number of homogeneous sub-regions. And finally, Carol et al. (2009) employed the Jacob’s solution (Jacob, 1950) to quantify the piecewise spatial variations of the hydraulic diffusivity of an aquifer located next to the River Ajo in Samborombon Bay wetland, Argentina. Theoretically speaking, the smaller thickness of the sub-zone leads to the higher accuracy of the solution from this approach. In contrast, the second approach to handle the heterogeneity of the aquifer is to assume that the hydraulic conductivity changed with spatial distance as described by a known function. This approach was reasonable because the hydraulic conductivity was found to linearly increase with the distance from the coastline in some alluvial aquifers (Montalto et al., 2006; Carol et al., 2009). To investigate the effect of aquifer heterogeneity on the groundwater-seawater interactions, Monachesi and Guarracino (2011) presented a one-dimensional analytical solution considering the linearly increasing hydraulic conductivity.

The primary focus of this study is to provide a new two-dimensional analytical solution of the groundwater flow in response to tidal fluctuations in a coastal aquifer-aquitard system considering the aquifer heterogeneity. This new solution may serve multiple purposes. For instance, the new solution can be used to inversely calculate the aquifer parameters based on the observed data (Pandit et al., 1991; Serfes, 1991). It provides quick calculation on hydraulic head without involving complicated and sometimes time-consuming numerical calculation. It can also be used to test the accuracy and robustness of a numerical simulation. Further, our solution provides a convenient way to investigate the dynamic groundwater response to various tidal fluctuations scenarios, in particular, investigating the propagation bias phenomenon directly. The propagation bias was found by Trefry and Bekele (2004) in Garden Island, Australia, and it represents the inconsistency between the theoretical calculation and the observed strong attenuation and small time lag between the head and tide fluctuations. Here, we also investigate the propagation bias phenomenon in detail using the new two-dimensional analytical solution.

**PROBLEM STATEMENT AND ANALYTICAL SOLUTION**

**Governing equation**

The aquifer-aquitard system composes of a leaky confined aquifer in the bottom, an aquitard (semi-permeable layer) in the middle and a top unconfined aquifer (Figure 1). The ocean is on the left side of the aquifer-aquitard system. The origin of the coordinate system is at the boundary of the aquifer and ocean interface (coastline). The x-axis is perpendicular to the coastline and toward the inland, the y-axis is parallel to the coastline, and the z-axis is vertical.

To analytically investigate the two-dimensional groundwater flow in response to tidal fluctuations, several
assumptions are inevitable. Firstly, the aquitard is homogenous and isotropic, and both aquifer and aquitard are with uniform thickness (Hantush, 1967). Secondly, the storage of the aquitard can be ignored. Thirdly, the tidal effect to the head in the unconfined aquifer is very small and can be negligible, compared with the effect of the confined aquifer (Jiao and Tang, 1999; Wang et al., 2013). Volker and Zhang (2001) pointed out that the second and third assumptions were too simplified, and might cause unacceptable errors when the leakance from the aquitard was greater than 1 day$^{-1}$. Jiao and Tang (2001) showed that the aquitard leakance was much smaller than 1 day$^{-1}$ for the actual applications, so errors coming from these two assumptions were negligible. White and Roberts (1994) observed that the tidal fluctuations in an unconfined aquifer were heavily damped and died out in less than 20 m from the coastal line, probably because of the large value of the specific yield of the unconfined aquifer. However, the tidal fluctuations may extend to more than 100 m from the coastline in the underneath confined aquifer. Therefore, the tidal effect to the head in the unconfined aquifer is assumed to be negligible.

Based on these assumptions, the problem can be described as follows:

$$
\frac{\partial h}{\partial t} + \frac{\partial}{\partial x}\left( BK_x \frac{\partial h}{\partial x} \right) + \frac{\partial}{\partial y}\left( BK_y \frac{\partial h}{\partial y} \right) + \alpha(h_0 - h) = \frac{\partial}{\partial x} \left( \frac{\partial h}{\partial x} \right), \quad t > 0, \quad x, y > 0
$$

(1)

where $h$ and $h_0$ are the hydraulic heads (L) of the confined and unconfined aquifers, respectively; $S$ is the storativity (dimensionless); $B$ is the thickness of the confined aquifer (L); $K_x$, $K_y$, and $K_z$ are the hydraulic conductivities (LT$^{-1}$) of the confined aquifer along the x-axis and y-axis, respectively; $\alpha = K_z / B_z$ is the aquitard leakance (T$^{-1}$) (Hantush, 1967; Sun and Zhan, 2006), where $K_z$ and $B_z$ are the hydraulic conductivity (LT$^{-1}$) and thickness (L) of the aquitard, respectively; $t$ is time (T); $x$ is distance from the coastline (L). Cardenas (2010) and Monachesi and Guarracino (2011) investigated the groundwater dynamics in response to tidal fluctuations using the following equation:

$$
K_x = K_{x0}(1 + bx),
$$

(2)

where $K_{x0}$ is the hydraulic conductivity $LT^{-1}$ of the aquifer at $x=0$, $b$ is the heterogeneity factor along the x-axis and it is the nonnegative increase rate of the hydraulic conductivity $LT^{-1}$. In this study, we also employ Equation (2) to describe the hydraulic conductivity along the x-axis. One notable point is that the hydraulic conductivity given by Equation (2) seems unrealistic, because it tends to be infinite when $x$ approaches infinity. However, Monachesi and Guarracino (2011) pointed out that the analytical solution was not affected by the values of hydraulic conductivity far away from the coastline.

With the influence of the combined effects of the gravitational forces exerted by the Moon and the Sun and the rotation of the Earth, the ocean surface rises and falls periodically. Although the ocean surface might be influenced by the non-astronomical forces, such as barometric events, the tides at the ocean-aquifer interface can still be described using a series of cosine functions. For the single-frequency tidal fluctuation, the boundary condition used in previous studies (Sun, 1997; Li et al., 2000; Tang and Jiao, 2001) is

$$
h(0, y, t) = h_{m0} + Ae^{-my} \cos(\omega t + \beta y + \theta),
$$

(3)

where $h_{m0}$ is the mean sea level (L); $A$ is the amplitude of the tide (L); $\omega$ is the tidal frequency (T$^{-1}$) and equals $2\pi/T$, where $T$ is the tidal period (T); $m$ is the damping coefficient of tidal amplitude (L$^{-1}$); $\beta$ is the separation coefficient (L$^{-1}$), which defines the change of phase with distance along the y-axis; and $\theta$ is the phase shift (dimensionless). The section on Analytical Solution for the Model with Multiple-Frequency Tidal Fluctuations will present the model of multiple-frequency tidal fluctuations. Equation (3) represents that the amplitude of the tide may decrease and the phase may vary with the wave propagated along the coastline (the y-axis), as shown in Figure 1.

For the boundary of the infinity, one has

$$
\left. \frac{\partial h}{\partial x} \right|_{x \to \infty} = 0.
$$

(4)

The boundary condition along the y-axis is free boundary (or the boundary is sufficiently far from the interest of domain, thus will not affect the flow). Equations (1–4) compose of the mathematical model describing a two-dimensional transient groundwater flow in a coastal aquifer affected by the single-frequency tidal fluctuation. This model is an extension of some previous models. For example, it reduces to the one-dimensional model of Monachesi and Guarracino (2011) by ignoring
the aquitard leakance and by assuming the tidal amplitude and the phase to be constant along the coastline (the y-axis). It becomes the model of Tang and Jiao (2001) for a homogeneous hydraulic conductivity \( b = 0 \). When the leakance from the aquitard can be ignored and \( b = 0 \), the new model becomes the same as the model of Sun (1997).

Analytical solution

The exact analytical solution of the mathematic model of Equations (1–4) can be obtained using the separation-of-variables method, which is widely employed to solve the problems related the groundwater-seawater interaction (Tang and Jiao, 2001). The solution is as follows:

\[
h(x, y, t) = h_{mld} + A e^{-my} \text{Re} \left[ \frac{H_0^{(1)}(ix) e^{i(\omega t + \beta y + \theta)}}{H_0^{(1)}(2\xi)} \right],
\]

where \( \text{Re} \) denotes the real part of the expression; \( H_0^{(1)} \) is the first-kind zero-order Hankel function; \( \xi = \frac{-T_x(x) + m i}{\omega} \), \( a = 2\xi \sqrt{1 + b^2} \), \( T_x = BK_x \), \( T_y = BK_y \) and \( i = \sqrt{-1} \) are the imaginary sign. \( T_x \) is the transmissivity of the aquifer at the ocean-aquifer interface, and it is equal to \( T_x \) at \( x = 0 \) where \( T_x = BK_x \). The details of the derivation are presented in Appendix. The attenuation \( (\chi) \) and the time lag \( (t_{lag}) \) of head fluctuations in the aquifer are the following:

\[
\chi(x, y) = \sqrt{E_r^2 + E_i^2},
\]

\[
t_{lag}(x, y) = -\frac{1}{\omega} \tan^{-1} \left( \frac{E_i}{E_r} \right),
\]

where \( E_r = \text{Re} \left[ \frac{H_0^{(1)}(ix) e^{-my}}{H_0^{(1)}(2\xi)} \right], \text{ and } E_i = \text{Im} \left[ \frac{H_0^{(1)}(ix) e^{-my}}{H_0^{(1)}(2\xi)} \right] \). \( \text{Im} \) denotes the imaginary part of the expression. One can see that a greater \( m \) could result in a smaller \( \chi \), which is dimensionless, but is independent of \( t_{lag} \).

Analytical solution for the model with multiple-frequency tidal fluctuations

The boundary condition Equation (3) involves a single tidal frequency. In reality, the tidal fluctuations may compose of multiple-frequency tides (Sun, 1997; Guo et al., 2010), and one has

\[
h(0, y, t) = h_{mld} + \sum_k A_k e^{-m_y} \cos(\omega_k t + \beta_k y + \theta_k),
\]

where \( A_k, m_k, \omega_k, \beta_k \) and \( \theta_k \) are the amplitude, the damping coefficient of tidal amplitude, the frequency, the separation coefficient and the phase shift of the \( k \)-th sinusoidal component of the tidal fluctuations, respectively. Following the solving procedure as the single-frequency tidal fluctuation, the exact analytical solution with the boundary condition of Equation (8) can be obtained as

\[
h(x, y, t) = h_{mld} + \sum_k A_k e^{-m_y} \text{Re} \left[ \frac{H_0^{(1)}(u_k) e^{(i\omega_k t + \beta_k y + \theta_k)}}{H_0^{(1)}(2\xi)} \right],
\]

where \( \xi^2 = -\frac{T_x(x) + m i}{\omega} \), \( u_k = 2\xi \sqrt{1 + b^2} \) and the definition of other parameters are the same as the ones in Equation (5). One notable point is that Equation (9) is obtained via the principle of superposition assuming that the groundwater fluctuation in the unconfined aquifer can be ignored, an assumption that has been widely used to solve the similar problems (Tang and Jiao, 2001; Sun et al., 2008).

COMPARISONS OF THE NEW SOLUTIONS WITH THE NUMERICAL SOLUTION

To deal with problems related to the groundwater flow in response to the tidal fluctuations using numerical solutions, one of the difficulties is how to handle the boundary condition at the coastline. Generally, the boundary condition at the coastline can be described by a summation of a series of sines or cosines functions, and the numerical solutions are sensitive to the spatial and temporal step sizes. COMSOL Multiphysics is a robust Galerkin finite-element software package that includes an environment component for modelling the type of governing equations of this study. The grid system of the COMSOL Multiphysics simulation composes of a series of triangles, and it is easy to refine the elements near the coastal boundary if necessary.

We have compared our analytical solution against the numerical solution for a case by setting \( S = 0.001, K_0 = 1 \text{ m/day}, K_y = 1 \text{ m/day}, B = 10 \text{ m}, b = 0.01 \text{ m}^{-1}, \alpha = 0.001 \text{ day}^{-1}, \theta = 0, A = 1 \text{ m}, m = 0.001 \text{ m}^{-1}, \omega = 5 \text{ day}^{-1} \) and \( \beta = 0 \). Tang and Jiao (2001) and Monachesi and Guarracino (2011) used similar parameter values in their studies. For the numerical solutions, an arbitrary initial head is set as the mean sea level (\( h_{mld} \)). Figure 2 shows the numerical solutions by COMSOL Multiphysics and the analytical solutions for different locations. Our analytical solutions agree very well with the numerical solutions except at the early time when the effect of the initial condition has not yet died out.

RESULTS AND DISCUSSIONS

Comparison of the new solution with previous analytical solutions

In this study, the new two-dimensional model of the groundwater flow in response to the tidal fluctuations considers the leakage from the overlying aquitard and the spatially variable hydraulic conductivity in the aquifer.
This model is an extension of some previous models (Jacob, 1950; Sun, 1997; Jiao and Tang, 1999; Tang and Jiao, 2001; Monachesi and Guarracino, 2011). For example, when the amplitude of the tide at the ocean-aquifer interface does not change with distance \((m = 0\) and \(\beta = 0\)), Equation (5) becomes

\[
H(x, y, t) = h_{ml} + Re\left[A \frac{H_0^{(1)}(\xi) e^{i(\omega t + \theta)}}{H_0^{(1)}(2\xi)}\right],
\]

where \(\xi = \frac{a - a_{sl}}{T_x}\), and the other variables are the same as Equation (5). If the values of leakance \(a\), the phase shift \(\theta\) and \(h_{ml}\) are all zeroes, Equation (10) becomes the solution of Monachesi and Guarracino (2011). When the aquifer is homogeneous \((b = 0)\), Equation (10) reduces to

\[
h = h_{ml} + Ae^{-\chi t} \sqrt{T_x m} \cos(\omega t - x \sqrt{T_x m} + \theta),
\]

which is the one-dimensional analytical solution obtained by Jacob (1950) for a homogeneous confined aquifer. By setting \(b = 0\), one could obtain the two-dimensional analytical solution of Tang and Jiao (2001) from Equation (5)

\[
h = h_{ml} + Ae^{-\chi t} \sqrt{T_x m} \cos(\omega t + \beta y + \theta - \frac{\omega S + 2\beta m T_x}{2\pi} x),
\]

where

\[
\chi = \exp(-x \frac{\omega S}{2T_x}) \quad \text{and} \quad t_{lag} = x \sqrt{\frac{S}{2\omega T_x}}.
\]

In a similar way, it is easy to prove that the new solution is an extension of some other analytical solutions, such as the two-dimensional analytical solution of Sun (1997) for a homogeneous confined aquifer without the aquitard leakance, and the one-dimensional analytical solution of Jiao and Tang (1999).

**Effects of the aquitard leakance on the propagation bias**

Propagation bias was found by Trefry and Bekele (2004) when investigating hydraulic head fluctuations subjected to asynchronous dual tidal propagation in Garden Island, Australia. The propagation bias refers to the inconsistency between the theoretical calculation and the observed strong attenuation and small time lag between the head and tide fluctuations (Trefry and Bekele, 2004), and it will be explained as follows.

Based on the analytical solution of one-dimensional model Equation (11), one has

\[
\chi = \exp(-x \frac{\omega S}{2T_x}) \quad \text{and} \quad t_{lag} = x \sqrt{\frac{S}{2\omega T_x}}.
\]

Obviously, \(\chi\) and \(t_{lag}\) are found to decline exponentially and increase linearly with distance from the ocean, respectively, and \(\ln(\chi)\) linearly changes with \(t_{lag}\). Trefry and Bekele (2004; see Figures 6c and 6f) found that the best-fitted curves of \(t_{lag}(2\pi)\)-\(\ln(\chi)\) were always above the observed data. They named this discrepancy propagation bias and used numerical simulations to show how vertical layering in aquifer properties could generate similar attenuation-lag phenomena. Sun et al. (2008) demonstrated that the leakance of the overlying confining layer could enhance the landward attenuation of the tidal head fluctuation and shortened the time lag, based on a one-dimensional quasi steady-state solution. To understand how the aquitard leakance affects the propagation bias in the two-dimensional heterogeneous aquifer system, three sets of leakance \((a)\) will be included in the following discussion: 0.1 day\(^{-1}\), 0.01 day\(^{-1}\) and 0.001 day\(^{-1}\). The other parameters are \(b = 0.01\) m\(^{-1}\), \(K_y = 1\) m/day, \(K_0 = 1\) m/day, \(B = 100\) m, \(S = 0.001\), \(A = 0.342\) m, \(m = 0.001\) m\(^{-1}\), \(\omega = 5\) day\(^{-1}\), \(\beta = 1.67 \times 10^{-6}\) m\(^{-1}\) and \(\theta = 0\). All these parameters refer to those used in previous studies (Sun, 1997; Tang and Jiao, 2001; Monachesi and Guarracino, 2011).

Figure 3 represents the change of the natural logarithm of \(\chi\) with \(x\) and \(y\) in the aquifer for different aquitard
leakance values. Three observations can be seen from this figure. Firstly, the amplitude of the hydraulic head decreases with $x$. Secondly, a larger leakance results in a faster attenuation of the amplitude of the hydraulic head with $x$. This observation is similar with the case previously investigated in the homogeneous aquifer-aquitard system (Rotzoll et al., 2008; Sun et al., 2008). This is because a larger leakance means a greater amount of cross-formation flow through the aquitard, resulting in a less role played by the ocean-land boundary effect. Thirdly, $\ln(\chi)$ are not linearly changes with $x$. This observation is different from one in the previous studies where $\ln(\chi)$-x follows a linear function (Trefry and Bekele, 2004; Huang et al., 2012). The other parameters are similar to the ones in the section on Effects of the Aquitard Leakance on the Propagation Bias, and they are $K_v = 1$ m/day, $K_0 = 1$ m/day, $B = 100$ m, $S = 0.001$, $A = 0.342$ m, $\alpha = 0.001$ day$^{-1}$, $m = 0.001$ m$^{-1}$, $\omega = 5$ day$^{-1}$, $\beta = 1.67 \times 10^{-6}$ m$^{-1}$ and $\theta = 0$. Figure 6 represents the attenuation of the head fluctuation ($\chi$) in the aquifer for different aquifer heterogeneity factor ($b$) values at $y = 50$ m. Several observations can be seen from this figure. Firstly, $\chi$ decreases with $x$, and the reason for this observation has been explained in the section on Effects of the Aquitard Leakance on the Propagation Bias. Secondly, the values of the amplitude are the same at $x = 0$ for different $b$ values. Thirdly, there is an intersection point between two...
lines of different \( b \) values along the \( x \)-axis, as shown in Figure 6. In the region from the coastline to the intersection point along the \( x \)-axis, a larger \( b \) value results in a faster attenuation of the hydraulic head with the landward distance. In contrast, a larger \( b \) value decreases the attenuation of the hydraulic head with the landward distance in the region beyond the intersection point.

Figure 7 represents the change of \( t_{\text{lag}} \) with \( x \) in the aquifer for different values of \( b \) at \( y = 50 \text{ m} \). One can obtain several observations from this figure. Firstly, the values of \( t_{\text{lag}} \) are the same at the coastline for different \( b \) values, and \( t_{\text{lag}} \) increases with \( x \). Secondly, \( t_{\text{lag}} \) decreases with the increasing \( b \) value for a fixed location away from the coastline. Thirdly, the slope of the curve of \( b = 0.001 \text{ m}^{-1} \) seems constant, and \( t_{\text{lag}} \) linearly changes with \( x \). For the other two cases, the slopes of the curves become smaller. The relationship between \( t_{\text{lag}} \) and \( x \) is non-linear, and seems to follow a power function, as demonstrated in Figure 7. Figure 8 shows the relationship between \( \ln(\chi) \) and \( t_{\text{lag}} \) at \( y = 50 \text{ m} \) for different \( b \) values. It is obvious that \( t_{\text{lag}} \) decreases with increasing \( b \) for the same \( \chi \). In other words, in addition to the aquitard leakance, the heterogeneity of the aquifer is also an important factor responsible for the propagation bias. Furthermore, the relationship between \( \ln(\chi) \) and \( t_{\text{lag}} \) follows the linear function when \( b \) is smaller than 0.001 m\(^{-1}\).

**SUMMARY AND CONCLUSIONS**

In this study, a two-dimensional analytical solution of the groundwater flow in response to tidal fluctuations in a heterogeneous aquifer-aquitard system is presented. The hydraulic conductivity is assumed to linearly increase with the distance from the coastline along the \( x \)-axis as \( K_x = K_0(1 + bx) \), where \( K_0 \) is the hydraulic conductivity of the aquifer at \( x = 0 \) and \( b \) is the heterogeneity factor of the aquifer. This newly developed analytical solution has been used to compare with the numerical simulation by COMSOL Multiphysics based on the Galerkin finite-element method, and results show that both solutions agrees with each other very well. The effects of the aquitard leakance and the aquifer heterogeneity factor on the propagation bias of the tidal fluctuations are analysed, where propagation bias refers to the inconsistency between the theoretical calculation and the observed strong attenuation \( (\chi) \) and small time lag \( (t_{\text{lag}}) \) between the head and tide fluctuations. The following conclusions can be drawn.

1. The factors resulting in the stronger propagation bias phenomena not only include the higher leakance of the overlying aquitard, but also include the larger heterogeneity factor of the aquifer.
(2) The relationship between $t_{lag}$ and the distance along the $x$-axis seems to be linear for the curve of $b = 0.001 \text{ m}^{-1}$ while it may obey a power function for the curves whose values of $b$ are greater than 0.01 $\text{ m}^{-1}$. The relationship between $\ln(\chi)$ and $t_{lag}$ is linear when $b$ is smaller than 0.001 $\text{ m}^{-1}$.

(3) There is an intersection line between two $\chi$ surfaces of different $b$ values. A larger $b$ results in a faster attenuation of the hydraulic head within the region from the coastline to the intersection line along the $x$-axis while decreasing the attenuation in the region beyond the intersection line.

ACKNOWLEDGEMENTS

This research was partially supported by Program of the National Basic Research Program of China (973) (No. 2011CB710600, 2011CB710602), National Natural Science Foundation of China (No. 41172281, 41372253) and the scholarship to Quanrong Wang from China Scholarship Council (CSC), Field Demonstration of Integrated Monitoring Program of Land and Resources in Middle Yangtze River Jianghan-Dongting Plain (No. 1212011120084), and study on Groundwater Resources and Environmental Problems in Middle Yangtze River Jianghan-Dongting Plain (No. 1212011121142). We thank three anonymous reviewers for their critical and constructive comments, and we thank Peter Knappett for checking the English.

REFERENCES


APPENDIX: DERIVATION OF EQUATION (5)

Similar with the previous studies, the analytical solution of Equations (1–4) will be derived by the separation-of-variables method. According to the form of boundary condition Equation (3), the analytical solution of Equation (1) can be in the form

$$H(x, y, t) = h_{ml} + AF(x)e^{-my_0}e^{i(\omega t + \beta y + \theta)},$$  \hspace{1cm} (A1)

where $H(x, y, t)$ and $F(x)$ are complex functions, and $h(x, y, t) = \text{Re}[H(x, y, t)]$. According to Equation (A1), Equations (1–4) can be rewritten as

$$S \frac{\partial H}{\partial t} = T_{so} \frac{\partial}{\partial x} \left[ (1 + bx) \frac{\partial H}{\partial x} \right] + T_x \frac{\partial^2 H}{\partial x^2} + \alpha(h_{ml} - H), \hspace{1cm} t > 0,$$

$$H(0, y, t) = h_{ml} + Ae^{-my_0}e^{i(\omega t + \beta y + \theta)},$$  \hspace{1cm} (A3)

$$\left( K_x \frac{\partial H}{\partial x} \right)_{x \rightarrow \infty} = 0,$$  \hspace{1cm} (A4)

where $T_{so} = BK_0, T_x = BK_y$. It is notable that the hydraulic head in the unconfined aquifer, $h_{ml}$, in Equation (1) is assumed to be constant and equal to the mean sea level, $h_{ml}$.

Substituting Equation (A1) into Equation (A2), one has

$$d \left[ (1 + bx) \frac{dF(x)}{dx} \right] - T_x (\beta + mi)^2 + \alpha + \omega Si \frac{dF(x)}{dx} = 0.$$

(A5)

Let $\varepsilon^2 = \frac{-T_x (\beta + mi)^2 - \alpha - \omega Si}{T_{so}b}$ and $u = 2\varepsilon \sqrt{1 + bx}$, Equation (A5) becomes

$$u \frac{d^2F}{du^2} + \frac{dF}{du} + uF = 0.$$  \hspace{1cm} (A6)

Equation (A6) is the zero-order Bessel equation, and its general solution can be written as (Abramowitz and Stegun, 1965):

$$F = C_1J_0(u) + C_2Y_0(u),$$  \hspace{1cm} (A7)

where $J_0$ and $Y_0$ are the first and second kinds of Bessel functions of the zero-order, and $C_1$ and $C_2$ are complex constants, which can be calculated using the boundary conditions. Using asymptotic expressions for the derivatives of $J_0$ and $Y_0$ (Arfken and Weber, 2005), and substituting Equations (A1) and (A7) into Equation (A4), one has

$$u(C_1J_1(u) + C_2Y_1(u))_{x \rightarrow \infty} = 0,$$

(A8)

Using the asymptotic forms of $J_1(u)$ and $Y_1(u)$ for infinite $u$ (Arfken and Weber, 2005), and the trigonometrical identities,

$$J_1(u) \approx \sqrt{\frac{2}{\pi u}} \cos \left( u - \frac{3}{4} \pi \right), \hspace{1cm} Y_1(u) \approx \sqrt{\frac{2}{\pi u}} \sin \left( u - \frac{3}{4} \pi \right),$$

(A9)

where $u = u_r + iu_i$ and $u_r$ and $u_i$ are real values. When $x \rightarrow \infty$, one has $u_r \rightarrow -\infty$ and $u_i \rightarrow \infty$ according to the definition of $u$. Therefore, one has

$$C_1 + iC_2 = 0.$$  \hspace{1cm} (A11)

Substituting Equation (A11) into Equation (A7) yields

$$F = C_1[J_0(u) + iY_0(u)] = C_1H_0^{(1)}(u).$$  \hspace{1cm} (A12)

where $H_0^{(1)}(u)$ is the Hankel function of the zero-order and the first kind (Abramowitz and Stegun, 1965). Considering the boundary condition Equation (A3), Equation (A1) becomes

$$C_1 = H_0^{(1)}(2\xi)^{-1}.$$  \hspace{1cm} (A13)

Substituting Equations (A7)–(A13) into Equation (A1), one can obtain the analytical solution

$$h(x, y, t) = h_{ml} + Ae^{-my_0} \text{Re} \left[ \frac{H_0^{(1)}(u)e^{i(\omega t + \beta y + \theta)}}{H_0^{(1)}(2\xi)} \right].$$  \hspace{1cm} (A14)